Project closing report

Following the original research plan, the project on Stochastic Cooperation was indeed organized into three main topics. I will now summarize the findings of each of these.

1. *Dynamic core concepts*

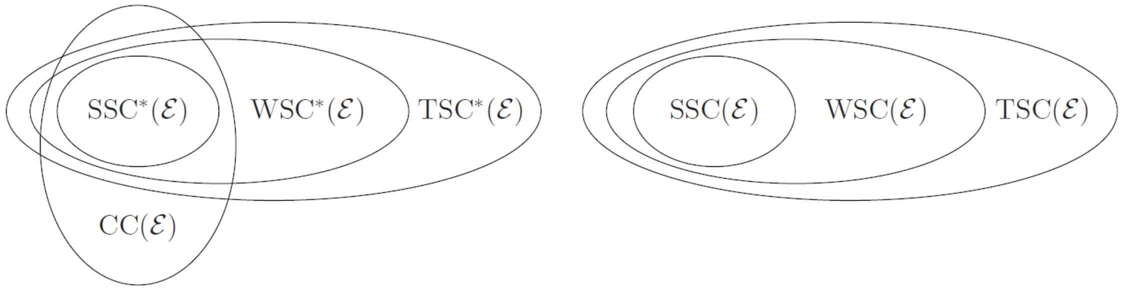
We analyzed all the dynamic core concepts already existing in the literature in our published paper:

Habis, Helga & Herings, P. Jean-Jacques, 2011. "**Core concepts for incomplete market economies**," Journal of Mathematical Economics, Elsevier, vol. 47(4-5), pages 595-609.

The key achievement of this part of the project is that we have managed to develop a new, unified structure of definitions. This structure highly facilitates the comparison of all these concepts.

We analyze dynamic cooperation in a two-period general equilibrium setting with possibly incomplete markets. General equilibrium theory captures the idea that markets are interrelated. The standard supply and demand partial equilibrium theory, which is looking at only one market at a time, cannot explain these dependencies of markets. An exchange economy in a dynamic general equilibrium model production decisions are assumed to be exogenously given. In each time-period the state of nature will determine the amount of goods and services, called commodities, the consumers will be endowed with. Consumers can exchange these commodities, and they can also trade with assets, which deliver commodities depending on the realized state of nature. Market incompleteness implies that players cannot make all possible binding commitments regarding their actions at future date-events.

We compare the core concepts in different market structure to highlight differences in outcomes of the solutions: for strongly complete markets, incomplete markets in finance economies, and incomplete markets in settings with multiple commodities. Even when markets are strongly complete, the Classical Core is argued not to be an appropriate concept. For the general case of incomplete markets the following graph shows the relationships we have found:



In general, the Strong Sequential Core is a subset of the Weak Sequential Core, which is a subset of the Two-stage Core and they are unrelated to the Segregated Core. The competitive equilibrium belongs to the Two-stage Core and to the Segregated Core but it may not belong to the other concepts. This property is perhaps less natural than it may seem as it is well-known that competitive equilibria are constrained suboptimal when asset markets are incomplete. It is therefore reasonable that this feature is recognized by an appropriate core concept; dynamic cooperation may overcome the inefficiencies of a competitive equilibrium in an incomplete markets setting. The Strong Sequential Core shares the weaknesses of the Classical Core, being a subset of it. Moreover, it is empty-valued for large classes of economies. All this leaves the **Weak Sequential Core** as the most satisfactory concept studied so far.

2. Stochastic TU games

In this part we have introduced uncertainty into the dynamic cooperative games by defining a new class of games called Transferable Utility Games with Uncertainty. Our findings were published in the following journal article:

Habis, Helga & Herings, P. Jean-Jacques, 2011. "**Transferable utility games with uncertainty**," Journal of Economic Theory, Elsevier, vol. 146(5), pages 2126-2139, September.

In transferable utility (TU) games, originating in the fundamental book of von Neumann and Morgenstern (1944), it is assumed that one player can losslessly transfer part of its utility to another player. This implies that irrespective of the division of the coalitional payoff, members of the coalition enjoy the same total utility. A TU-game is defined by the set of players and a characteristic function which assigns to each coalition its worth. The solution of the game is an allocation of the worth of the grand coalition.

While the majority of real-life decisions and economic problems involve tradeoffs over time and uncertainty, dynamic cooperation has gotten limited attention. Introducing dynamics raises many new conceptual questions. It is not straightforward how to extend the classical definition of the core to this setting. Specifically, there are various ways in which one might formulate the notion of a profitable deviation.

In an environment with no commitment possibilities only self-enforcing allocations can be deemed stable. This consideration leads to the notion of credibility. A deviation of a coalition is credible, if no sub-coalition ever has a credible counter-deviation from it. While the set of deviations and credible deviations coincide in the static game, as it is shown by Ray (1989), the distinction makes a crucial difference in the dynamic case. Incorporating credibility into the definition of the core, leads to a dynamic solution concept, the Weak Sequential Core. The Weak Sequential Core of the game is the set of feasible allocations for the grand coalition from which no coalition ever has a credible deviation. Furthermore this concept was found to be the most appropriate dynamic core solution.

We study an environment with two time-periods and uncertainty regarding the state of nature in the last period; just like it is the case with our afore-mentioned example of the estate division problem. Depending on the state of nature in the last period, a particular transferable utility game is played. We define this setting as a dynamic transferable utility game with uncertainty, or briefly a TUU-game. The introduction of uncertainty is a natural and very important extension of TU-games. We adopt the modified definition of the Weak Sequential Core to this stochastic setting and provide its characterization in the spirit of Kranich, Perea, and Peters (2005).

The main purpose of the paper is to show **non-emptiness** of the Weak Sequential Core. It is well-known that the core of the static TU-game is non-empty if and only if it is balanced (Bondareva (1963) and Shapley (1967)). In the dynamic setting, both Kranich, Perea, and Peters (2005) and Predtetchinski, Herings, and Perea (2006) give requirements for non-emptiness for special cases, but there is no general result in the literature so far. Predtetchinski, Herings, and Perea (2006) show that it is possible to give examples where all the standard assumptions are satisfied and the Weak Sequential Core is empty. The Weak Sequential Core can also be empty in the simpler TUU environment. We use our characterization result to prove the existence of the solution set. It turns out that convexity of the stage-games, together with continuity of the utility functions, is sufficient for the non-emptiness.

We use these theoretical results in two follow-up papers applied to bankruptcy problems:

Habis, Helga, 2012. "**Sztochasztikus csődjátékok - avagy hogyan osszunk szét egy bizonytalan méretű tortát? [Stochastic bankruptcy games. How can a cake of uncertain dimensions be divided?]**," Közgazdasági Szemle (Economic Review - monthly of the Hungarian Academy of Sciences), Közgazdasági Szemle Alapítvány (Economic Review Foundation), vol. 0(12), pages 1299-1310.

Helga Habis & P. Herings, 2013. "**Stochastic bankruptcy games**," International Journal of Game Theory, Springer, vol. 42(4), pages 973-988, November.

A famous example which leads to a convex game is the bankruptcy game, originating from the Talmudic estate division problem. It is easy transform the original problem into a TUU-game, by assuming that both the value of the estate to divide and the claims of the players may be uncertain; see for instance our example above. We show that most of the best-known allocation rules, such as the Proportional rule or even the Talmudic rule, are unstable in this stochastic game. The Constrained Equal Awards rule is the only one, which belongs to the Weak Sequential Core of the TUU-game.

3. *Externalities*

In this part we introduce a new class of transferable utility games, where we allow for dynamics, uncertainty and externalities as well, and propose an extended notion of the core as a solution to this game which relaxes its restrictive assumptions. Our main results are published in the following article:

Habis , Helga & Csercsik, Dávid, 2012. "**Cooperation with Externalities and Uncertainty**," Networks and Spatial Economics, 2014. Sept

In this paper we will combine two directions of research, and allow for the presence of uncertainty and externalities at the same time by introducing **partition function form game with uncertainty** (hereafter PFU-game). A PFU-game consists of two time periods, 0 and 1. In period 1 one out of a finite number of states of nature may materialize and conditional on the state, the players are involved in a particular partition function form game. An outcome therefore specifies a payoff-partition configuration conditional on each possible state of nature. A utility function is then used to assign a utility level to each profile of state-contingent payoffs.

We are interested in the appropriate definition of the core of a PFU-game. In this setting coalitions (forming some partition) are allowed to form in both time periods. Stability requires that a suggested allocation cannot be blocked by any coalition at any period, i.e. both before and after the resolution of uncertainty. Binding agreements are not natural in such an environment, thus only self-enforcing agreements should be allowed for. These considerations lead to the concept of the Sustainable Core. Extending the characterization of the Weak Sequential Core, we say that an allocation belongs to the Sustainable Core only if conditional on the state of nature it belongs to the Recursive Core of the PFF-game related to that state, and moreover there is no coalition (forming some partition) in period 0 that can propose state-contingent Recursive Core elements of the game restricted to that coalition, which gives its members higher utility.

An important application of the PFU-game is the electrical energy transmission network, where consumers and generators need to find a stable allocation of power in- and outlets, taking into account possible line failures. Csercsik (2011) show that both negative and positive externalities exist in a static electrical energy transmission network game of generators and consumers, and therefore suggest the Recursive Core as a solution concept. Here we extend their example by the possibility of line failures in the grid, which creates the uncertainty. We show that the Sustainable Core can be used to solve this extended game.