Final report

1 Background

An important cause for the complexity of equilibrium, kinematical and dynamical bifurcations of elastic bar structures is the interplay between various displacement modes. The coupling between bending, shear, torsion and extension can often result in a non-trivial set of bifurcations. It has been established that even relatively simple bar structures can possess very complex post-critical behaviour. One of the main aims of our research was to discover the effects of coupling between displacement modes through various examples.

The investigation of simple mechanical or mechanistic models, like the search for the equilibrium paths building up global bifurcation diagrams of bars and bar structures or dynamical explanation of various phenomena, has received great recent interest also in biology. DNA conformations, cytoskeleton, fungal hyphae are all examples where a mechanical analysis turned out to be useful. Therefore, during the project, we often took the investigated examples from the field of modelling biological systems using a mechanistic point of view. Besides obtaining results in engineering, we managed to achieve new scientific results in relation to biological systems.

We planned to publish our results in international journals and at conferences. We published 14 journal papers (11 with impact factor) and there are 3 more papers submitted to journals with impact factor. One chapter we wrote appeared in a book. Other 6 papers appeared in conference proceedings. There was one PhD dissertation defended, and one more finalized during the course of the project. All in all, 27 publications were triggered by the support of this project.

2 Results

The references in (brackets) refer to the corresponding elements of the publication list.

In order to investigate bifurcation diagrams, first we intended to improve existing numerical techniques. Global equilibrium paths of rod structures in the post-critical state are usually represented in the global representation space (GRS) which gives a unique description of all equilibrium configurations. However, the GRS, even for simple cases like the planar web of links, is already higher dimensional. The Scanning Simplex Algorithm is a powerful and robust numerical method to approximate solutions of non-linear problems possessing a one-dimensional solution set in the GRS. The previously known form of the scanning simplex algorithm could only find 1-dimensional objects, equilibrium paths in the GRS. However, in many applications the solutions within the GRS form 2D objects, surfaces. We developed an extended version of the scanning simplex algorithm, which makes it capable of finding

2 or higher dimensional solution sets, e.g., surfaces in the GRS of non-linear problems. A further enhancement of the method was that we implemented it as a parallelized computer code using OpenMP environment. Our code is capable of using multiple cores of the computer. The applied parallelization scheme is applicable for further research, and we indeed used it in our further studies. We applied our method for the calculation of the equilibrium surfaces of the clamped-clamped hemitropic rod with two load-parameters, and calculated the perturbation of the equilibrium paths of an elastic web of links. Results in the GRS are visualized using the POVRay computer system, which is capable of visualizing large number of surface elements. This allowed for a qualitative analysis of the resulting solution surfaces. The results are published as a journal paper (Németh, 2014). We note that due to the ban of purchasing IT infrastructure, we were not able to obtain a powerful computer capable of running CUDA parallel codes during the first three years of the project, which largely delayed us to achieve the set goals.

We investigated a generalized discrete elastica including both bending and shear (a generalization of the Hencky bar–chain model), and revealed its possible link with non-local beam continua. Shear is introduced by either the Engesser and the Haringx approach, both supported by physical arguments. Buckling and post-buckling of the systems are analysed, the buckling loads are analytically calculated, and the post-buckling behaviour is numerically studied for large displacement. Also, we built non-local Timoshenko-type beam models from the continualization of the discrete systems. The fundamental buckling loads of the non-local elastica models are analytically obtained, and their first post-buckling paths are numerically computed. The results are summarized in a journal paper submitted for publication (Kocsis *et al*, 2017.)

The elastic linkage and the elastic web of links are two simple discrete models that approximate the behaviour of bars with finite bending and shear stiffness, respectively. We showed that in a combined model, in the case of the bi-elastic web of links, the critical load parameter values depend linearly on the ratio of the bending stiffness to the total stiffness of one level of the web. We showed that the bi-elastic web of links is a discrete model of Csonka's beam. A journal paper was published containing these results (Németh, Kocsis, 2014). In a related investigation, using topological reasoning, we looked for the necessary and sufficient conditions to find stable packing of rectangular blocks by a web of strands. By stable we mean that the strands cannot be removed from the block without cutting or straining them. It can be shown that applying three or less closed strands whose segments are parallel to the edges of the block can never be stable, it always provides a finite mechanism. We found, however, that loops with oblique segments can be found in pairs such that the resultant binding forms a first-order infinitesimal mechanism that can be stiffened by pre-stress. For some geometries, we also found configurations with optimal strand length (Kovács, 2015). In a further step, it has been shown that if there is a hexahedral solid with all its angular defects being multiples of $\pi/2$, then there exists another solid in its arbitrary close neighbourhood that can be bounded by a set of equidistant, mutually intersecting strings that form an orthogonal weaving over the respective surface (Kovács, 2016).

We investigated the in-plane buckling of plates possessing a periodic set of holes. Based on the kinematics of a simple, discrete web of links model, we explained the observed checkerboard-like pattern formation observed during buckling experiments carried out at the University of Manchester. Despite the simplicity of our model, the estimation of the buckling load was close to the experimentally measured value. The results were presented at a conference in collaboration with researchers from Manchester (S Willshaw, T Mullin, Gy Károlyi: Elastic buckling of 2D cellular structures. In: M Stockmann, J Kretzschmar (Eds.): 10th Youth Symposium on Experimental Solid Mechanics, Chemnitz, Germany, 2011, pp. 119-120.) However, no further experimental results could be obtained from the group at the University of Manchester, hence further publication of these results did not follow.

Long drill strings used e.g. in oil or geothermal energy industry show very complex dynamical behaviour. During the often used rotary drilling technique, the several kilometres long drill string is rotated around its longitudinal axis by a torque applied on the surface while the drill bit exploits the rock. The complex interplay of bending vibrations of the drill string, axial motions resulting in bit jump, stick-slip like torsional vibrations results in a difficult dynamical problem. We investigated a simplified, discrete model of this complex problem. We extended a previous drill bit model, based on delayed dynamics, to include more degrees of freedom to investigate their effects on stability of motion. We also investigated the effects of resonance enhanced drilling on the drilling process. Previous results showed that longitudinal vibration of the drill bit results in a faster and more stable drilling. Using an enhanced, more accurate model of the impact of the drill bit on rock formation, we found that the advantages of resonance enhanced drilling (faster and more stable drilling) remain unaltered using this model (Ajibose *et al*, 2012).

In the field of the dynamics of structures turning over and breaking, we investigated the vibrations of tall structures with nonlinear constitutive relations. We found that kinematic constraints lead to variable stiffness, implying that the dynamics separates into a fast and a slow component. In this regard, we analysed the dynamics of a beam on block-and-tackle suspension system (Németh and Kocsis, 2015). In this structure, a suspension cable of the beam creates the constraint, which separates the structural behaviour into a stiffer one with a faster dynamics and a more compliant one with a slower dynamics via repeated slacking and stressing of the suspension system. The methodology for the numerical handling of the continuous model is demonstrated on a few examples of free or forced vibrations. From the results, a journal paper and a conference paper was published (Németh and Kocsis, 2015, Kocsis *et al*, 2015). Similar investigations were carried out for cracked beams. The existing cracks in the beam result in a piecewise linear material behaviour by the opening and closing of the cracks, similarly to the active and passive states of the suspension system in the case of suspended structures. We analysed both a discrete mass-and-springs, and a continuum model of prestressed bars, and developed a method for the calculation of the energy-dependent natural frequencies of both the multi-degree-of-freedom system and the continuum. From the results, two conference papers were published (Lengyel and Németh, 2015 and 2016).

We also carried out investigations concerning the effects of an aircraft impacting a structure of high stiffness. The standard model, invented by Riera in 1968, assumes that during the impact of the aircraft modelled as simple rigid-plastic rod the impacted structure can be considered rigid. With this assumption, the reaction forces acting on the structure can be computed and then they can be applied on the structure to compute the stresses, strains, displacements, global damage (e.g., collapse or turning over) and local damage (e.g., perforation or penetration) of the structure. This way the impact problem is decomposed into the break of the aircraft and its effects on the structure. However, the question arises whether and under what conditions the structure can be considered rigid during the impact. Based on comparisons with finite element models, we found that only for very robust structures can this assumption be accepted. The first results appeared as a Scientific Student's Association project (LE Laczák: Repülőgép ütközése vasbeton szerkezettel, BME Természettudományi Kar, TDK dolgozat, 2012) and were presented at a conference (Laczák, 2013). We also analysed the global and local effects of hard and soft impacts by developing finite element models of reinforced concrete structures in order to find the effects of different parameters (wall thickness, concrete quality, impact velocity). Models were compared to previous real full-size and medium scale experiments and good agreement of results was found (Laczák and Károlyi, 2013 and 2014). Using numerical and analytical techniques, we found a single parameter to characterize the global effects of the impact of a uniform aircraft into robust structures. This dimensionless parameter, which we call *damage potential*, is the ratio of the aircraft's initial kinetic energy to the work required to break its whole length. These results were first presented at a conference (Laczák and Károlyi, 2015). The role of the damage potential was first identified in a wide-range parametric survey of the Riera model. For the simplest case of a uniform aircraft and a rigid target, analytical results confirmed that the only relevant dimensionless parameter combination is the damage potential that affects the course of the impact. Numerically the same result was confirmed for more general cases as well. These results are accepted as a journal paper (Laczák and Károlyi, 2017 (International Journal of Non-linear Mechanics)). Then the same behaviour was confirmed in a more refined finite element model, the corresponding results are submitted for publication (Laczák and Károlyi, 2017 (International Journal of Solids and Structures)). A science popularization paper also appeared in this field (Laczák and Károlyi, 2016 (Építés-építészettudomány)). We also analysed the local effects of the impact, which occur when the missile is more rigid than the target structure. Both for the depth of penetration and for the minimum target wall tickness required to avoid perforation, we derived new formulae that depend on a dimensionless impact parameter including the parameters of both the aircraft and the target. We find a very good agreement of our formulae to both experimentally measured values and numerically obtained results using a finite element code. The results are published as a journal paper (Laczák and Károlyi, 2016 (Periodica Polytechnica)). The results of these investigations provide the basis for a PhD thesis (Laczák, 2017).

We also investigated a biology-oriented problem related to the motor protein myosin II, responsible for muscle contraction. We proposed to extend the previous standard model of myosin II, consisting of a head domain and a lever arm, by adding an internal lever arm to the model. We analysed this extended model under the combined effect of an external load and thermal fluctuations and found the behaviour consistent with experimental data

(Bibó *et al*, 2013). We also shed new light on the debate among biologists whether the power stroke of myosin II was a ratchet mechanism or a relaxing mechanism following a conformational change. We found that the two mechanisms coexist which can explain the contradiction between the measured efficiency of the molecule in macroscopic and molecule-level experiments. We suggest that the main part of the work during muscle contraction occurs at the very start of the power stroke during a very short time period, caused by a ratchet mechanism, which is followed by a relaxation period having a sensory role. A journal paper with these result was submitted (Bibó *et al*, 2017). Based on the myosin related results, a PhD dissertation was successfully defended (Bibó, 2013).

In the general field of the proposal, we achieved progress in several more problems not originally included in the work plan.

First, we investigated the strains and stresses of photo-elastomer beams under the combined effect of external mechanical load and illumination. It was previously known that in the case of bending such photo-elastomer beams, there can be more than one stress free layers in the cross-sections. During the project, we obtained an analytical result on the number of stress free layers in the cross-sections of the beam. The actual number depends on the parameters of the problem. The results were presented as a Scientific Student's Association project (Z Ábrahám: Fotoelasztikus anyagú gerendák feszültségei és alakváltozásai, BME Építőmérnöki Kar, TDK dolgozat, 2012). A journal article was also published from the results (Ábrahám and Károlyi, 2015).

The second field not included in the work plan concerns the dynamic relaxation of a simple magnetic pendulum. According to standard textbooks, this system would be a typical example for transiently chaotic mechanical systems. However, as we pointed out, the only long-term behaviour in this system is reached when motion stops in an equilibrium state due to dissipation, hence no long-time (either regular or chaotic) motion is possible, prohibiting the standard form of transient chaos to exist in this system. We named this novel type of behaviour *doubly transient chaos*. We found that standard chaos parameters take on a different role in this system, for example, the well-known escape rate, instead of being constant, increases exponentially with time in the case of the magnetic pendulum. The results were published as a journal paper (Motter *et al*, 2013).

Third, we numerically identified filamentary fractal structures in blood flow in real cerebral aneurysms, and pointed out that the most robust parameter to characterize the fractality, which hopefully can find clinical relevance, is the information dimension (Závodszky *et al*, 2015 and 2016). A science popularization paper also appeared in this field during the second year of the project (Károlyi, 2013).