

Report on the research supported by OTKA Grant K 100796.

The PI (**A. Némethi**) published the following articles since the beginning of the present OTKA project K 100796, some of them by himself, some of them with collaborators from the OTKA grant (**T. László, B. Sigurdsson, G. Pintér, J. Bodnár**), and some of them with other collaborators.

(1) Generalized monodromy conjecture in dimension two, (joint manuscript with W. Veys), *Geometry and Topology* **16**(1) (2012), 155–217.

(2) Equivariant classes of matrix matroid varieties (joint paper with L.M. Fehér and R. Rimányi), *Commentarii Math. Helvetici* **87** (2012), 861–889.

(3) Some meeting points of singularity theory and low dimensional topology, Proc. of the Deformation Theory Conference, Budapest, 2008. Bolyai Society Mathematical Studies **23**, Springer-Verlag (2013), 109–162.

(4) Spectrum of plane curves via knot theory, (joint manuscript with M. Borodzik), *Journal of LMS*, II Ser. **86** (1) (2012), 87–110.

(5) The lattice cohomology of $S^3_{-d}(K)$, (joint manuscript with F. Román), in *Zeta Functions in Algebra and Geometry, Contemporary Mathematics* **566**, Amer. Math. Soc., Providence, RI, 2012, 261–292. (Proc. of 2010 Mallorca conference).

(6) A counterexample to Durfee’s conjecture, (joint manuscript with D. Kerner) *Mathematical Reports of the Acad. of Soc., The Royal Soc. of Canada*, **34** (2) (2012), 50–64.

(7) The cohomology of line bundles of splice-quotient singularities, *Advances in Math.*, **229**(4), 2503–2524 (2012).

(8) Heegaard Floer homologies for (+1) surgeries on torus knots (joint manuscript with M. Borodzik), *Acta Math. Hungarica* **139**(4) (2013), 303–319.

(9) Hodge–type structures as link invariants, (joint manuscript with M. Borodzik), *Annales de L’Institute Fourier* **63**(1), 269–301 (2013).

(10) The ‘corrected Durfee’s inequality’ for homogeneous complete intersections, (joint manuscript with D. Kerner), *Math. Zeitschrift* **274**, Issue 3–4 (2013), 1385–1400.

(11) Links of singularities up to regular homotopy, (joint manuscript with A. Katanaga and A. Szűcs), Proceedings of 12th International Workshop on Real and Complex Singularities, 2012 San Carlos, Brasil; *Journal of Singularities* **10** (1914), 174–182.

(12) Ehrhart theory of polytopes and Seiberg–Witten invariants of plumbed 3-manifolds, (joint manuscript with T. László), arXiv:1211.2539, *Geometry and Topology*, **18** (2014), 717–778.

(13) On the semicontinuity of the mod 2 spectrum of hypersurface singularities, (joint manuscript with M. Borodzik and A. Ranicki), *Algebraic Geometry* **24** (2015), 379–398.

(14) Reduction theorem for lattice cohomology, (joint manuscript with T. László), *Int. Math. Res. Notices* **2015**, Issue 11 (2015), 2938–2985.

(15) The geometric genus of hypersurface singularities (joint manuscript with B. Sigurdsson), arXiv:1310.1268, to appear in *JEMS*.

(15) Holomorphic arcs on analytic spaces (joint manuscript with J. Kollár), *Inventiones Math.* 200 issue 1 (2015), 97–147.

(16) The Hodge spectrum of analytic germs on isolated surface singularities (joint manuscript with M. Borodzik), *J. Math. Pures Appl.* 103 (5) (2015), 1132–1156.

(17) Immersions associated with holomorphic germs (joint with G. Pintér), arXiv:1404.2853, to appear in *Comm. Math. Helv.*

(18) Lattice and Heegaard Floer Homologies of Algebraic Links, (joint manuscript with E. Gorsky), to appear in *IMRN*. (It is the revised version of ‘Poincaré series of algebraic links and lattice homology’ arXiv:1301.7636).

- (19) Morse theory for manifolds with boundary, (joint manuscript with M. Borodzik and A. Ranicki), arXiv:1207.3066, to appear in *Algebraic & Geometric Topology*.
- (20) Links of plane curve singularities are L-space links, (joint manuscript with E. Gorsky), arXiv:1404.2853, to appear in *Algebraic & Geometric Topology*.
- (21) Lattice cohomology and rational cuspidal curves, (joint manuscript with J. Bodnár), arXiv:1405.0437.
- (22) Durfee-type bound for some non-degenerate complete intersection singularities, (joint manuscript with D. Kerner), arXiv:1405.7494.
- (23) A generalized FKG-inequality for compositions, (joint manuscript with D. Kerner), arXiv:1412.8200.
- (24) Durfee's conjecture on the signature of smoothings of surface singularities, (joint manuscript with J. Kollár, with an appendix by T. de Fernex), arXiv:1411.1039.
- (25) Seiberg–Witten invariant of the universal abelian cover of $S^3_{-p/q}(K)$, (joint manuscript with J. Bodnár), arXiv:1505.03005.
- (26) Codimension 2 embeddings, algebraic surgery and Seifert forms, (joint manuscript with M. Borodzik and A. Ranicki), arXiv:1211.5964.
- (27) Discriminant of the ordinary transversal singularity type (joint manuscript with M. Kazarian and D. Kerner), arXiv:1308.6045.
- (28) The cohomology of the tangent bundles on resolution of weighted homogeneous surface singularities (joint paper with T. Okuma), submitted.
- (29) On the set of L–space surgeries for links, (joint manuscript with E. Gorsky), arXiv: 1509.01170.
- (30) Links of rational singularities, L-spaces and LO fundamental groups, arXiv:1510.07128.

A large part of the manuscripts deal with the lattice cohomology associated with the topological type of normal surface singularities (5, 8, 12, 14, 15, 18, 21, 25, 30). They treat different aspects of the theory, or, the lattice cohomology is used as the main tool in the proof of the results. The main applications target superisolated singularities (5,21,25), hypersurfaces with non-degenerate principal part (as 15), or surgery 3-manifolds. These include the computation of the Seiberg-Witten invariant of the corresponding singularity links (using surgery formulae or the fact that the Seiberg-Witten invariant is the normalized Euler characteristic of the lattice cohomology).

In (12) with T. László we found a deep connection of the lattice cohomology (in particular, of the Seiberg-Witten invariants) and of the topological zeta series of the resolution lattice of a surface singularity with (equivariant) Ehrhart theory (this is the theory of lattice points counting in polytopes).

In (14) we analyse major properties of the lattice cohomology of negative definite plumbed 3-manifolds. The main result establishes a theorem, which reduces the rank of the lattice in the definition of the cohomology theory to the number of ‘bad vertices’ of the plumbing graph. Several applications are listed, mostly related with the topological series (zeta function) associated with the graph: we prove that the multivariable periodic constant of the reduced series still can be identified with the Seiberg-Witten invariant.

Collaboration with E. Gorsky (18,20,29) provides several results connecting the L-space properties of surgery 3-manifolds (along algebraic links) with the algebraic properties of the corresponding links. In (20) we prove that any algebraic link is an L-space link, in (20) we recover the Knot Heegaard Floer homology of algebraic links via their algebraic Hilbert series (using also the theory of Orlik-Solomon algebras), and also as the coefficients of the motivic Poincaré series.

In (15) we prove that for several hypersurface singularities the geometric genus is the Euler characteristic of the path-0-lattice cohomology. The main cases include germs with Newton nondegenerate principal parts, and superisolated singularities (the second one is pathological from the point of view of the Seiberg-Witten Invariant Conjecture).

Several articles written jointly with M. Borodzik (and A. Ranicki) target the connection of (mixed) Hodge theory (motivated by the properties of the spectrum associated with holomorphic germs and their topological connections) with general link theory (9, 13, 16). E.g., for holomorphic germs defined

on normal surface singularities we define a weak version of the Seifert matrix and we relate it with the Hodge spectrum. The semicontinuity of the Hodge spectrum is also proved using equivariant signature type invariants.

In (15) we introduce several spaces of holomorphic arcs associated with a local analytic germ. The work is motivated by the famous Nash conjecture targeting formal arcs, and their properties. We establish the holomorphic version, and we determine the connected components of the space of short holomorphic arcs. The main cases include the quotient singularities and normal surface singularities.

In (17) we relate the topological notion of the Smale invariant associated with immersion of the 3-sphere into the 5-sphere with holomorphic invariants of maps from the complex 2-space to 3-space. We characterize via analytic invariant the Smale invariant (whenever it is realized by analytic maps), in this way we answer an old question of Mumford.

The collaboration with D. Kerner targets the famous conjecture of Durfee (weak and strong versions) (10, 22, 23) on the negativity of the signature of the Milnor fibers. We proved that the bound for the strong inequality in the original conjecture of Durfee for (non-hypersurface) complete intersections is not valid, and we formulated a new conjectural inequality, what we verified for homogeneous complete intersections.

In collaboration with J. Kollár (24) we proved several versions of the Durfee conjecture, including the negativity of the signature of the Milnor fiber for any isolated hypersurface singularity.

In the recent manuscript (30) I proved the old conjecture, which says that a singularity link (of a normal surface singularity) is L-space (in the sense of the Heegaard Floer theory) if and only if it is the link of a rational singularity.

Some additional publications of **J. Bodnár**:

Bodnár, J.: Classification of rational unicuspidal curves with two Newton pairs, arXiv:1504.07875, 2015, to appear in *Acta Math. Hungar.*

Bodnár, J., Celoria, D., Golla, M.: A note on cobordisms of algebraic knots, preprint, arXiv:1509.08821, 2015.

Bodnár, J., Celoria, D., Golla, M.: Cuspidal curves and Heegaard Floer homology, arXiv:1409.3282, to appear in *Proc. London Math. Soc.*, 2014.

The research attacks the connection between the analytical and topological invariants of normal surface singularities and plane curve singularities. He examined the local types of plane curve singularities occurring on complex projective plane curves (targeting the famous problem of topological classification of cuspidal projective plane curves), and the Seiberg–Witten invariants of Dehn surgeries along connected sum of algebraic knots.

An additional publications of **Á. Gyenge**:

Á. Gyenge: Enumeration of diagonally colored Young diagrams, arXiv <http://arxiv.org/abs/1510.02677>

The aim of the research of Á. Gyenge is to describe the Hilbert scheme parametrizing the zero-dimensional subschemes of some basic classes of surface singularities. He plans to connect the topology of these moduli spaces to the topology of the (link of) the singularity using representations of vertex algebras. This would give a two-dimensional analog of a conjecture of Oblomkov and Shende, proved recently by Maulik, as well as a singular version of the AGT correspondence. He wrote the following manuscript (but several other long manuscripts will be available soon):

Some additional publication of **T. László**:

László, T., Szilágyi, Zs.: On Poincaré series associated with links of normal surface singularities, arXiv:1503.09012 (2015).

This proves a decomposition theorem for the topological Poincaré series, as a multivariable generalization of the ‘polynomial and negative degree parts’ decomposition in the one-variable case considered by Braun and Némethi, and in the two-variable case proved by László and Némethi in earlier

articles. The polynomial part defines a polynomial invariant for negative definite plumbed 3-manifolds, which would be the ‘polynomial generalizations’ of the Seiberg–Witten invariant.

The publications of **A. Stipsicz** in this period:

Heesang Park, A. I. Stipsicz: Smoothings of singularities and symplectic surgery arXiv:1211.6830
J. Symplectic Geom. 12 (2014), no. 3, 585–597.

Ozsváth, P.; Stipsicz, A.; Szabó, Z.: Knot lattice homology in L-spaces, arXiv:1207.3889, *Journal of Knot Theory and its Ramifications*, to appear

Ozsváth, P.; Stipsicz, A.; Szabó, Z.: Knots in lattice homology. *Comment. Math. Helv.* 89 (2014), no. 4, 783–818.

J. Bowden, D. Crowley, A. Stipsicz: On the topology of Stein fillable manifolds in higher dimensions, I, *Proc. London Math. Soc.*, 109 (2014) 1363–1401.

J. Bowden, D. Crowley, A. Stipsicz, The topology of Stein fillable manifolds in high dimensions, II. *Geom. Topol.* 19 (2015), no. 5, 2995–3030.

H. Park, A. Stipsicz, Symplectic 4-manifolds via symplectic surgery on complex surface singularities. *Bull. Korean Math. Soc.* 52 (2015), no. 4, 1213–1223.

P. Ozsváth, A. Stipsicz, Z. Szabó, A spectral sequence on lattice homology. *Quantum Topol.* 5 (2014), no. 4, 487–521.

A short description of these articles:

By extending lattice homology for (appropriate) knots in plumbed 3-manifolds, in a joint project with Ozsváth and Szabó, Stipsicz proved the isomorphism of lattice homology and Heegaard Floer homology for further classes of 3-manifolds. These results provide further evidence to the conjectured isomorphism of the two theories (conjecture of Némethi).

In a sequence of papers (with J. Bowden and D. Crowley) higher dimensional manifolds are examined from the point of view if they admit Stein fillable contact structures. The obstruction found by Bowden–Crowley–Stipsicz is sufficient to show that many manifolds and almost contact structures do not admit Stein fillable representatives, and in particular one can show that a connected sum can be Stein fillable without the components admitting Stein fillable structures.

In a joint project with H. Park, Stipsicz proved an extension of symplectic surgery, relying on open book decompositions of Milnor fillable contact structures. As an application of this new method, interesting symplectic 4-manifolds near the Bogomolov–Miyaoaka–Yau line have been constructed.